U.S. Department of Justice
Federal Bureau of Investigation

SPECIAL AGENT EXAM PREPARATION MANUAL

FBI SPECIAL AGENT SELECTION PROCESS

ATTENTION APPLICANT:

THIS MANUAL IS THE PROPERTY OF THE FEDERAL BUREAU OF INVESTIGATION - YOU ARE REQUIRED TO RETURN IT ON THE DAY YOU ARE SCHEDULED FOR TESTING. PLEASE DO NOT MARK IN AND/OR DEFACE THIS MANUAL.

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Chapter 1
Introduction

The Purpose of this Book

The purpose of this workbook is to assist individuals who are preparing to take the FBI Special Agent Examination. For an overall description of the Special Agent Examination, individuals should refer to the Applicant Information Booklet, FBI Special Agent Selection Process. The present workbook covers parts of the Cognitive Ability Test with a primary focus on a review of the mathematics procedures and concepts that are included in that test.

Much of the material in this workbook covers very basic concepts so applicants for the FBI Special Agent position should use this workbook as much or as little as they feel is appropriate to prepare for the test. This workbook is offered free of charge from the FBI to give applicants a realistic preview of the test content and to facilitate any preparation that individuals may wish to undertake.

This book is organized according to the general areas of mathematics that are included in the FBI Cognitive Ability Test. The individual sections are ordered according to the knowledge and skills required to solve basic arithmetic, algebra and geometry problems. For example, the book begins with basic arithmetic with whole numbers. This section is followed by one involving basic arithmetic with fractions. Each subsequent section builds upon the previous one to some degree.

How to Use This Book

There are two ways to use the mathematics sections of this book.

1. Begin working where you feel you need the most help. If you have difficulties with problems in that section, move to an earlier section.

2. Take the short diagnostic exam to help you decide where to begin. The diagnostic exam is located in Appendix A. Remember, if you chose to take the diagnostic exam, it was not designed to measure your full range of abilities. Nor was it designed to directly mirror the actual test content. Rather, it was designed to give you a relative measure of your current strengths and weaknesses.

In either case, after you have reviewed the concepts and procedures in the front part of the book you can work sample problems and review solutions in Appendix B. Most of the sample problems are set up as multiple choice problems, but some problems in the basic math and algebra sections require you to generate the answer -- something you should do even when taking multiple choice exams.

One final note, if you feel that you need additional practice beyond what is offered in this workbook, several different types of workbooks and study guides for college and graduate school admissions tests are available at local bookstores.

Chapter 2
Test Preparation and Test Taking

Preparing to Study

There are two basic purposes for studying -- to learn new concepts or skills, and to recall and reinforce skills that you haven't used in a while. In either case, effective study requires:

1) a focus of attention on the material, and

2) active thinking and "participation" with the material.

Thinking and learning require concentration, and you can not concentrate when you are faced with numerous distractions. Since mathematics requires more focus than many other subjects, it is especially important to minimize distractions, both internal and external. You will not have quality study time if you are worrying about other things. Do your best to put all the other things on your mind aside while you study. Once you get yourself into the right frame of mind, make sure your external environment is conducive to study. If there are too many distractions where you are trying to study (e.g., home) then go to a library, public park, or other place where you can be relatively free from distractions.

Study Tips

Educational researchers have spent decades investigating how people learn, but have not been able to translate this knowledge into a universal set of study principles. One of the reasons is that people, especially adults, vary considerably in their learning styles. Despite the lack of a common set of guidelines, below are some general guidelines that can help.

1) Learn by doing

Effective learning is an active process — you learn better when you are practicing the skills you are acquiring. In the case of math that means learning concepts and procedures and then doing problems that involve those concepts and procedures.

2) Monitor your progress

Effective learning requires feedback — you learn better when you can correct errors and reinforce successes. In the case of math that means looking at the results of your work and determining where and why you made errors, and reviewing the steps of correct solutions to reinforce those successes.

3) Spread your study time out, don't cram

Effective learning takes time — quality time. You learn better when you can spread the process out over a comfortable period of time. Unfortunately, there is no easy way to know in advance how long it will take to learn new skills or refresh old ones. In order to insure that you will have adequate study time, start early. You can always make use of extra time at the end of your study time, but you can't create time when you meet a deadline.

Test Taking Tips

Test taking, like studying, is an active process. If you have studied effectively, then test taking should be a continuation of the study process. The major differences, however, are that you:

a) must conform to specific time constraints

b) can not look back to the reference pages for information, and

c) can not obtain immediate feedback on your performance.

These three factors often work together to make the test taking environment very stressful. A moderate amount of stress is good, it helps us to be alert and attentive. Too much stress, however, harms performance. The test taker's task, then, is to try to minimize the stress by taking advantage of the full range of skills he/she brings to the testing situation. The following tips will help you achieve that goal by making the best use of the available time and drawing upon your knowledge and information in the test.

1. Know that this is a timed test so mentally allocate your time so you can pace yourself through the test.

2. Read through the entire question to make sure you know what is being asked. If you fail to fully read a test item you run the risk of solving the wrong problem and having to do additional work.

3. Make a decision on whether or not to attempt a problem solution.

   If you know how to solve the problem, proceed. If you think you know, but are not sure, spend a little more time trying to find a solution. If you do not see an approach don't get flustered. Skip the item and return to it later. If you really don't know an answer, generate a guess based on an elimination of alternatives. *There is no penalty for guessing on the actual exam.*

4. On items on which you have difficulty try to break the problem into smaller steps

5. Don't make the test harder than it is.

   Many people assume that, by definition, a math problem has to be hard. As a result, they often "look" and find a harder problem than the one they have been asked to solve. This is especially true of word problems. There is no simple way to identify the difficulty of a test item. Just read each problem carefully and apply some of the methods described in this book.

6. Evaluate the plausibility of your answer.

   One of the beauties of mathematics is that is provides a set of procedures that allow us to derive precise answers to difficult problems. This is great when the correct procedure is identified and all the steps are carried out without error. The application of an incorrect procedure, or a relatively small error in one step can, however, lead to a very different answer. As a test taker, you must be prepared to "step back" from your work and assess the plausibility of your answer -- even if it matches one of the response alternatives. Test designers often develop incorrect response items on the basis of common errors.

   When you generate an answer to a problem evaluate its plausibility. In some cases you may have no idea, but in other cases you may recognize that your answer does not make sense (e.g., a 1500 lb. man, or a passenger car that goes from Chicago to New York in 2 hours).

Chapter 3
Number Systems and Basic Arithmetic

This chapter is designed for those individuals who feel they need to refresh their knowledge of number systems and the basic mathematical operations for different number types. The table below provides an index of the various topics.

Numbers and Properties of Numbers

Mathematics is built around different number systems with each system being defined according to a set of properties. For example,

The system of *real numbers* is divided into two sets, *rational numbers* and *irrational numbers*. The set of *rational numbers* is further divided into *integers* (whole numbers such as 0, 1, 3, and -5.) and *fractions* (e.g., 1/4, 2/5). *Irrational numbers* are those that can not be represented as integers or fractions (e.g., π). For most applications, however, we simplify mathematics by transforming any irrational number into a nearby rational number. We also will refer to the *natural numbers* (or counting numbers). These are the positive integers (e.g., 1, 2, 3...459....690).

Notation and Operations

In order to get the various number systems to work for you it is necessary to have a set of arithmetic operations. Common symbols for arithmetic operations include:

Arithmetic Operations

Operation	**Symbol**	**Example**
Addition	+	$3 + 4 = 7$
Subtraction	-	$7 - 3 = 4$
Multiplication	x	$3 \times 4 = 12$
	()	$3(4) = 12$
	()()	$(3)(4) = 12$
	·	$3 \cdot 4 = 12$
	*	$3 * 4 = 12$
Division	÷	$12 \div 4 = 3$
	——	$\frac{12}{4}$ - 3
	/	$12/4 = 3$

Composite and Prime Numbers

Composite numbers: A composite number is one that can be represented as the product of two or more natural numbers other than itself and 1. For example:

$45 = 5 \cdot 9$ or $45 = 5 \cdot 3 \cdot 3$

The numbers that are multiplied together are called whole number factors.

Prime numbers: A prime number is one whose only whole number factors are 1 and itself. For example:

$$67 = 1 \cdot 67$$

Fractions

Fractions are *rational numbers* that are written in the form of an indicated quotient of two whole numbers:

$$\frac{a}{b} \quad \text{or} \quad a/b$$

where a and b are integers. The number in the top is referred to as the numerator and the number in the bottom is referred to as the denominator. The denominator cannot be 0 (zero) since division by zero is undefined.

In most instances a fraction represents a part of a whole with the numerator designating the number of parts and the denominator designating the total number of parts in the whole. For example,

Most problems encountered with performing arithmetic on fractions involve

a) the need for a common (i.e., same) denominator to perform some, but not all, computations, and

b) the lack of understanding the effects of the computations. The following "rules" should clarify these issues.

1) You need a common denominator for the addition and subtraction of fractions but not for the multiplication or division of fractions.

For example, when you have a common denominator like, $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ you simply add the two values in the numerator and keep the same denominator. The same situation occurs when you want to subtract two fractions, $\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$ In this case, however, you subtract the smaller number from the larger and keep the same denominator.

When you have unlike denominators you must transform one or both of the fractions to create two fractions with a common denominator. This transformation can be done in one of two ways. One way is to multiply both the numerator and the denominator by the same number. This is equivalent to multiplication by 1 which changes the form of the fraction but not its value. The other way is to divide the numerator and denominator by the same number. The number you chose in either case will be one that will yield the value you want for the common denominator.

For example, with the problem $\frac{2}{4} + \frac{1}{8} =$ you can multiply the fraction $\frac{2}{4}$ *by* $\frac{2}{2}$ which is equivalent to multiplication by 1 and therefore does not change the value. This will produce $\frac{2}{4} x \frac{2}{2} = \frac{4}{8}$. At this point you will substitute that result into the original problem and carry out the necessary addition. $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$

2) The addition and subtraction of fractions yields the same effects as with whole numbers, in that adding two fractions yields a result that is larger than either of the two original fractions, subtracting one fraction from another yields a result that is smaller than the larger of the two original fractions.

Whole number addition: $3 + 1 = 4$ Fraction addition: $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

Whole number subtraction: $3 - 1 = 2$ Fraction subtraction: $\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$

Multiplication and division by a proper fraction (less than 1) has the opposite effect from those operations on whole numbers. Multiplication of whole numbers yields an answer that is greater than the two original numbers and dividing one whole number by another yields a number that is smaller than the one that was divided. For fractions this relationship is reversed. Multiplication of two fractions yields a smaller fraction and division of a number by a proper fraction yields a larger number.

Whole number multiplication: $3 \cdot 4 = 12$ Fraction multiplication: $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$

Whole number division: $12 \div 6 = 2$ Fraction division: $\frac{2}{4} \div \frac{1}{4} = \frac{8}{4}$

Decimal Numbers

A decimal number is shorthand for fractions whose denominator is 10, 100, 1000, or some power of 10. (For a description of power see a later section in this chapter). Thus, 0.35 is equal to 35/100. Decimal numbers provide another way to represent values between whole numbers. When performing operations with decimal numbers, the main thing to keep track of is the placement of the decimal point. It is also important to note that decimal numbers behave in the same way as fractions.

When you add two decimal numbers, the result will be larger than either of the two original numbers. $.35 + .35 = .70$

When you subtract two decimal numbers, the result will be smaller than the largest of the two original numbers. $.70 - .35 = .35$

When you multiply a number by a decimal number, the result will be smaller than the original number. $.70 \cdot .35 = .245$

When you divide one decimal into another the result will be larger than the original number. $.70 \div .35 = 2$

Rounding

One concern when dealing with decimals involves the process of rounding when you need to reduce the number of decimal places in your answer. The general rule for rounding is to round up when the last decimal place is greater than 5 and round "down" when the last decimal place is less than 5. When the last decimal place is exactly 5 you make the choice based on the value of the next to last decimal place. For even numbers you round up, and for odd numbers you round down. For example,

.346 becomes .35 when you round up

.343 becomes .34 when you round "down"

.345 becomes .35 because 4 is an even number

Negative Numbers

Negative numbers like -1 or -3 are below or less than zero. The following examples demonstrate the various effects of negative numbers on addition and multiplication.

Addition with negative numbers can affect both the sign and the absolute value of the result.

Addition with two positive numbers: $3 + 3 = 6$

Addition with one positive and one negative number: $3 + (-3) = 0$

Addition with two negative numbers: $(-3) + (-3) = -6$

Multiplication and division with negative numbers effects the sign but not the absolute value of the result.

Two positive numbers:	3 * 3 = 9	9/3 = 3
One positive and one negative number:	3 * (-3) = -9	9/-3 = -3
Two negative numbers:	(-3) * (-3) = 9	-9/-3 = 3

Powers and Roots

Powers and roots are shorthand techniques for writing quantities. Powers are numbers that have exponents which represent a multiplicative relationship. For example,

5^2 is another way of writing the number 25.

In order to determine the value of a power you multiply the number by itself *n* times where *n* is the value of the exponent. For example, 3^3 is the same as 3 * 3 * 3 or 27.

Roots are essentially the opposite of powers. Thus $\sqrt{25}$ (square root of 25) is 5 since 5 squared is 25. Other roots are denoted by a number in front of the radical for example, the cube root is written as $\sqrt[3]{27}$ and is equal to 3 since 3 cubed is 27.

Laws of Exponents

The following table provides a general set of rules regarding the effect of arithmetic on exponents.

Operation	Example
$X^a X^b = X^{a+b}$	$(5^2)(5^3) = 5^5$ or 3125
$(X^a)^b = X^{ab}$	$(5^2)^3 = 5^6$ or 15625
$(XY)^a = X^a Y^a$	$(5 * 6)^3 = (5^3)(6^3)$ or 27000
$\frac{X^a}{X^b} = X^{a-b}$	$\frac{5^3}{5^2} = 5^{3-2}$ or 5

Chapter 4
Applied Arithmetic

This chapter is designed for those individuals who feel they need a refresher on the concepts shown in the following table.

Concept/Procedure	Example	Page Number
Percentages	On the way to Chicago, IL from Washington, D.C., a distance of 800 miles, you stop in Cleveland, OH. You have traveled 450 miles. What percentage of the trip have you completed?	14
Rates and Ratios	A plane travels from Washington, D.C. to Pittsburgh, PA, a distance of 350 miles, in 1 hour and 15 minutes. What was the plane's average speed?	15
	A student's grade is based on 4 quizzes, 1 midterm, and 1 final exam. The midterm and the final exam are each equivalent to 2 quizzes. What proportion of the final grade is determined by the midterm?	
Averages	The starting line-up of a basketball team includes a 5'10" point guard, a 6'4" shooting guard, a 6'8" small forward, a 6'10" power forward, and a 7'1" center. What is the average height of the starting line-up?	16
Weighted Averages	A car dealership sold 50 new cars at an average of $5,000 per car and 43 new cars at an average of $12,000 per car. What was the average sale price for all cars sold?	16

Percentages

A percentage shows the relationship between two quantities, a part and a whole, in terms of 100%. The basic calculation is:

$$\frac{\text{Part}}{\text{Whole}} * 100\%$$

For example, on the way to Chicago, IL from Washington, D.C., a distance of 800 miles, you stop in Cleveland, OH. You have traveled 450 miles. What percentage of the trip have you completed?

Set up problem	$\frac{450 \text{ miles}}{800 \text{ miles}} * 100\%$
Step 1. Divide 450 by 800	.5625 * 100%
Step 2. Multiply and write in terms of a percentage	56.25%

A variation of the percentage calculation is to determine the amount of change in a quantity. The basic formula is:

$$\frac{\text{Amount of Change}}{\text{Original Whole}} * 100\%$$

For example, a shirt was originally priced at \$30.00 and has been reduced by \$3.00. What was the percent reduction?

Set up problem	$\frac{\$3.00}{\$30.00} * 100\%$
Step 1: Divide reduction by original amount	.10 * 100%
Step 2: Multiply and write as a percentage	10%

This same basic procedure can be used to compute percent increases as well. Using basic algebra it is also possible to solve for any of the unknown quantities when given the other two. For example, given a percentage increase and the original price, you can determine the amount of the increase.

A proportion is computed in the same way as the percentage with the exception that you do not multiply by 100%.

For example, Jim weighed 250 lbs. prior to his diet. He lost 25 lbs. in 4 months. What proportion of his original weight did Jim lose?

Set up problem $\frac{25 \text{ lbs.}}{250 \text{ lbs.}}$

Step 1: Divide part quantity by whole quantity .10

Rates and Ratios

Both percentages and proportions are based on ratios of like quantities (e.g., lbs. to lbs.). A rate is a quantity that shows the relationship between one quantity and a unit measure of a second different quantity (e.g., miles per hour, cents per pound).

$$\frac{\text{Quantity \#1}}{\text{Quantity \#2}}$$

A plane travels from Washington, D.C. to Pittsburgh, PA, a distance of 350 miles, in 1 hour and 15 minutes. What was the plane's average speed?

In the case of computing average speed the formula is:

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

Set up problem $\text{Average speed} = \frac{350 \text{ miles}}{1.25 \text{ hours}}$

(Note that you must translate time into a standard measure like minutes or hours)

Step 1: Divide number of miles by the number of hours and write the result in terms of miles per hour.

280 miles per hour

Averages

An average, also referred to as the *arithmetic mean*, is computed with the following formula:

$$\text{Average} = \frac{\text{Total of Terms}}{\text{Number of Terms}}$$

For example, the average of the numbers 12, 14, 10, and 4 is computed as follows:

Set up problem $$\text{Average} = \frac{12 + 14 + 10 + 4}{4}$$

Step 1. Add terms in the numerator $$\text{Average} = \frac{40}{4}$$

Step 2. Divide numerator by denominator $$\text{Average} = 10$$

It is also possible to determine other relationships involving an average through simple algebraic operations (discussed in Chapter 6). For example, given an average, the number of terms, and all but one term, it is possible to determine the value of the missing term.

Weighted Average

A weighted average is computed when the terms contribute differentially to the total. For example, a teacher may give three 100 point exams during a semester but want the final exam to be weighted twice as much. In order to determine a given student's average score the teacher would use the following formula:

$$\text{Average} = \frac{\text{Test \#1} + \text{Test \#2} + 2(\text{Test \#3})}{\text{Total of weights}}$$

Since two tests are assigned weights of 1 and the third test a weight of 2, the total of the weights is 4.

$$\text{Average} = \frac{70 + 85 + 2(77)}{1 + 1 + 2} \quad \text{or} \quad \text{Average} = 77.25$$

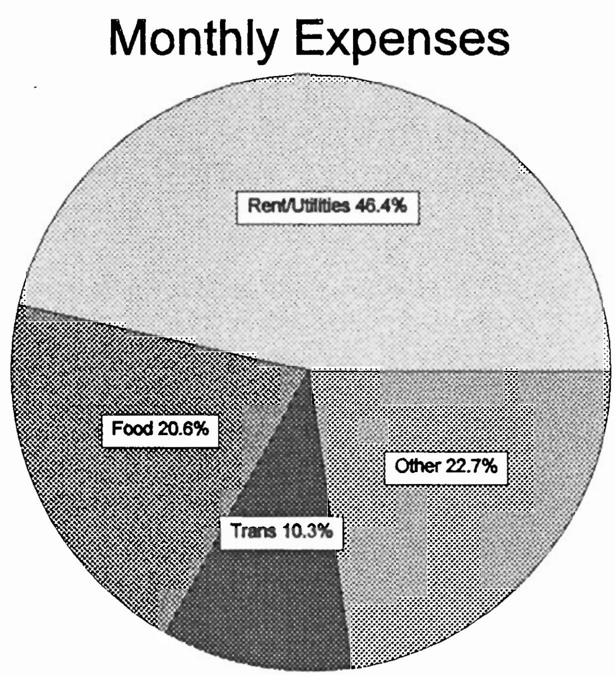
Chapter 5
Graphs, Charts, and Tables

This chapter is designed for those individuals who feel they need a refresher on the concepts shown in the following table.

Concept/Procedure	Page Number
Pie Charts	17
Bar Graphs	18
Line Graphs	19
Tables	20

Pie Charts

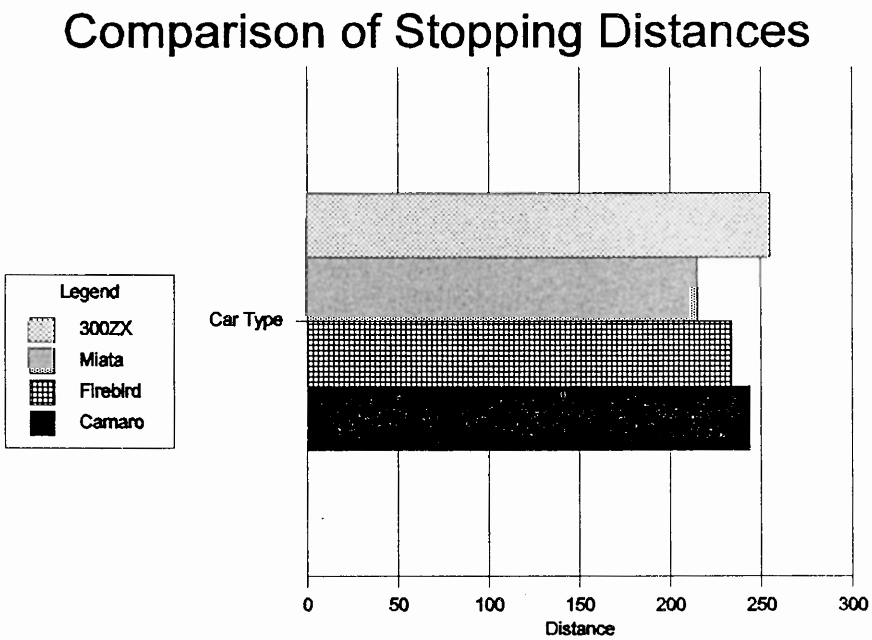
Graphs and charts are pictorial representations of quantitative information. Pie charts are round pictorial displays of proportions/percentages of a whole. For example, the following pie chart shows the distribution of the percentages of household income for various budget items.



The information in the pie chart can be used in a variety of ways. In addition to reading the information directly from the chart (e.g., 46.4% of the monthly expenses go to rent and utilities), it is possible to derive other relationships. For example, the expenditures for food is twice that for transportation (20.6% versus 10.3%).

Bar Graphs

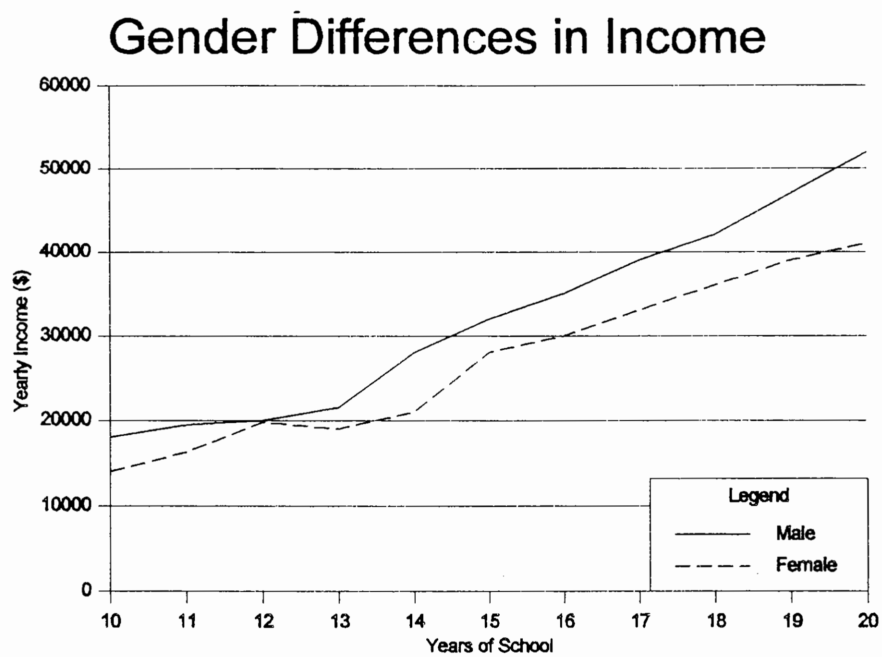
Bar graphs show the relationships between *quantitative* and *qualitative* information. For example, the following bar graph shows the relationship of stopping distance (a quantitative measure) for different types of cars (qualitative measure) from 40 miles per hour.



As with the pie chart, it is possible to read information directly from the bar graph regarding both qualitative relationships (e.g., the Miata stops in the shortest distance) and quantitative relationships (e.g., the 300ZX takes over 250 feet to stop).

Line Graphs

A line graph typically shows the relationship between two quantitative measures. The following line graph shows the relationship of schooling to yearly income for males and females.



The line graph above enables the user to look at a variety of relationships for either males or females separately, as well as comparisons between the two groups. For example, both groups show increasing income as a function of increased education, but males show greater return on their schooling. At a specific level of schooling (e.g., 15 years) males make about $32,000 and females about $28,000.

Tables

Tables can be of various types and complexities. Tables consist of columns (vertical) and rows (horizontal). The following table show the same information as the line graph above.

Years of Education	Income	
	Male	Female
10	$18,000	$16,500
11	$19,000	$17,000
12	$20,000	$20,000
13	$21,500	$19,500
14	$26,500	$20,500
15	$32,000	$28,000
16	$34,000	$30,000
17	$38,000	$31,500
18	$42,000	$35,000
19	$48,000	$38,000
20	$52,000	$41,000

Chapter 6
Algebra

This chapter is designed for those individuals who feel they need a refresher on the concepts shown in the following table.

Concept/Procedure	Example	Page Number
Algebraic Expressions	$2(x^2 + 2x - 3)$	21
Algebraic Equations	$x^2 + 4x + 4 = (x + 2)(x + 2)$	24
Algebra Word Problems	If John has twice as many apples as Bill and Bill has 4 apples, how many apples does John have?	25

Algebra is basically an extension of arithmetic in which numbers can be represented by other symbols (usually Roman or Greek letters).

Algebraic expressions

An algebraic expression is a symbolic form involving constants, variables, mathematical operations, and grouping symbols. The common grouping symbols are parentheses ( ), brackets [ ], and braces { }. Variables are usually represented by single letters such as x or y, but a variable can be expressed by letter combinations or even words.

$8 + 7$ is an example of an algebraic expression with two constants and one mathematical operation.

$3(4 + 5)$ has three constants and two operations.

$3(x + 6)$ has two constants, one variable, and two operations.

Two or more algebraic expressions joined by addition (+) or subtraction (-) signs are called terms; two or more algebraic expressions joined by multiplication are called factors.

$3x + x$ has 2 terms.

$3x + 5(x + 3)$ has two terms, the second term has two factors, 5 and $(x + 3)$.

Order of operations in algebra

One of the most common operations in algebra is to simplify an expression. When you simplify an expression you are trying to reduce the number of terms by performing mathematical operations.

In order to simplify correctly, it is necessary to sequence mathematical operations correctly. The two rules of simplifying are:

1. Simplify from the inside out. Start with the innermost symbols of a grouping (i.e., those most deeply enclosed in grouping symbols) and perform that operation. Once that operation is completed, move to the next innermost grouping.

2. Multiplication and division take precedence over addition and subtraction.

To simplify the expression $24 - 3 \cdot 6$

multiply $3 \cdot 6$ and rewrite the expression as $24 - 18$

subtract 18 from 24 and rewrite the expression as 6.

To simplify the expression $12 + 3(4 + 3)$

add $4 + 3$ and rewrite the expression as $12 + 3 \cdot 7$

multiply $3 \cdot 7$ and rewrite the expression as $12 + 21$

add $12 + 21$ and rewrite the expression as 33.

Expressions with variables

Simplifying expressions with variables follows the same rules as just discussed. For example, given the following expression: $3(3x + 3x + 6)$

First, start from the inner most term and perform the legal operations. In this case you would add $3x + 3x$ and get $6x$. Rewriting the expression you get:

$3(6x + 6)$

Since you are unable to add $6x$ and 6 (You can not add a number with a variable to any unlike number) you now carry out the final step by multiplying 3 times $6x$ and 3 times 6.
The result is: $18x + 18$.

What would happen if the original expression were $3x(3x + 3x + 6)$? The operations would be essentially the same with the exception of the final step. Performing the inner most operations would yield:

$$3x(6x + 6)$$

Now you would multiply $3x$ times $6x$, and $3x$ times 6. In the first case the result would be $18x^2$ and in the second case the result would be $18x$ giving a final answer of:

$$18x^2 + 18x.$$

Notice that the x associated with the 3x has been distributed to the terms inside the parentheses. This is an example of the *distributive property*. In its general form it is represented as:

$$a(b + c) = ab + ac$$

Note that the "a"," b"," c" in this expression can represent complex terms like $18x^2$.

One thing that makes algebra easier to remember is the ability to recognize common forms or templates that occur during the expansion or factoring of expressions. Three of the most common templates are:

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

$$(a - b)(a + b) = a^2 - b^2$$

These templates are very useful tools to use when you are trying to solve algebraic equations.

Division with variables can create some problems, but it follows the same rules as multiplication.

Given the expression $\dfrac{4x + 6}{2}$

You would divide each of the terms in the numerator by 2.

In effect the expression becomes: $\dfrac{4x}{2} + \dfrac{6}{2}$

Notice that the 2 in the denominator was distributed to each term in the numerator. When you carry out the division the expression becomes $2x + 3$. You can check this by substituting a value of 3 for x and solving both expressions. In the first instance you get 12 + 6 divided by 2 which equals 9. When 3 is substituted into the expression $2x + 3$ you get the same answer.

In more complicated expressions, however, you need to be careful not to divide too soon. For example when evaluating: $\frac{3(3x + 3)}{3}$ you get an incorrect answer if you carry out the division before you carry out the multiplication in the numerator. The correct procedure is to multiply 3 times 3x and 3 times 3 to yield (9x + 9) in the numerator. Carrying out the division now yields 3x + 3. Notice that this is the same as you get if you "cancel" the 3's in the numerator and denominator.

Algebraic Equations

Algebraic equations provide the means to fully utilize the powers of algebra. The rules for handling equations are the same as those for expressions. The only difference is that an equation has two sides. For example, $x^2 + 2 = 18$ is a simple algebraic equation with a single variable. In order to solve a single variable algebraic equation you need to isolate the variable on one side of the equation. This is done by a series of transformations that follow one basic rule: Anything you do to one side of an equation you must also do to the other side. This is a fairly simple concept, but is sometimes difficult to accomplish because of the complexity of the equation. In order to balance transformations, it is important to remember the basic rules of algebra.

In the example above, $x^2 + 2 = 18$ can be transformed as follows:

(1) $x^2 + 2 = 18$
(2) $x^2 + 2 - 2 = 18 - 2$
(3) $\sqrt{x^2} = \sqrt{16}$
(4) $x = 4$

Step (1) is the original equation. Since we want to isolate the variable x, at Step (2) we subtract 2 from each side of the equation. This leaves us with $x^2 = 16$. At Step (3) we take the square root of each side of the equation which leaves us with the resulting $x = 4$. We can check this result by substituting this value of x back into the original equation; 16 + 2 = 18.

A more complicated example would be as follows:

(1) $2x^2 + 50 = 4x^2$
(2) $2x^2 + 50 - 50 = 4x^2 - 50$
(3) $2x^2 - 4x^2 = 4x^2 - 4x^2 - 50$
(4) $-1(-2x^2) = -1(-50)$
(5) $\frac{2x^2}{2} = \frac{50}{2}$
(6) $\sqrt{x^2} = \sqrt{25}$
(7) $x = 5$

Again, the goal is to isolate the variable x on one side of the equation. The original equation at Step (1) is first transformed by subtracting 50 from each side. At Step (3) we get all the terms with x on the left side by subtracting $4x^2$ from each side. At Step (4) each side is multiplied by -1 to make both terms positive and at Step (5) each side is divided by 2 . (Notice that these two steps could be combined by dividing by -2). Finally, at Step (6) we take the square root of each side which leaves us with x = 5. Substituting back into the original equation we get: 2(25) + 50 = 4(25) or 50 + 50 = 100. This verifies that we have performed the various steps correctly.

Algebra Word Problems

Algebra word problems are basically equations described in terms of words. One of the keys to developing an accurate solution is to identify what the problem is seeking for a solution, the unknown. The unknown then becomes the variable in your equation. For example,

> A school with 300 children has twice as many boys as girls.
> How many girls are in the school?

The unknown in this problem is the number of girls. Let x represent the number of girls. We know that there are twice as many boys as girls so we can represent the number of boys as $2x$. We can now construct the following equation that captures the necessary information in the problem.

$$x + 2x = 300$$

(i.e., girls + boys = 300)

We can combine x and 2x and rewrite the equation as 3x = 300 Dividing each side by 3 we get x = 100. So the number of girls in the school is 100. In checking the solution we see that the number of boys is 200, which is twice the number of girls.

It is not uncommon to read an algebra word problem and find more than one unknown. Don't worry. Use the given information to help you look for other relationships that might lead you to a solution. For example,

> A mobile home has two water tanks. Tank A can be filled at a rate of 10 gallons per minute and Tank B can be filled at a rate of 15 gallons per minute. Water is pumped into both at the same time until they are both filled (a total of 300 gallons). How much water has been pumped into Tank A?

The unknown in this problem is the amount of water pumped into Tank A which we can represent with the variable a. We also know that a is some proportion of 300 and that the other part is the volume of Tank B, b. So we can write an equation of $a + b = 300$. The problem with this equation, however, is that we have two unknowns. What we do know is the fill rate of each tank so we can determine how long it took to reach 300 gallons by adding together the two fill rates (10 and 15) to get a total fill rate of 25 gallons per minute. Next, we can determine how long it took to pump 300 gallons by dividing 300 by 25. The result shows us that it took 12 minutes to fill the tank. At a rate of 10 gallons per minute for 12 minutes Tank A would end up with 120 gallons; $10 * 12 = 120$. Tank B, on the other hand, would end up with $15 * 12$ or 180 gallons. As we originally stated the total would be $120 + 180 = 300$.

Chapter 7
Geometry

This chapter is designed for those individuals who feel they need a refresher on the concepts shown in the following table.

Concept/Procedure	Page Number
Types of polygons	27
Features of common polygons	28
Perimeter of Polygons	31
Area of Polygons	33

Many of the procedures in geometry are based around polygons -- many sided figures. The table below shows the number of sides and names of common polygons.

Types of Polygons

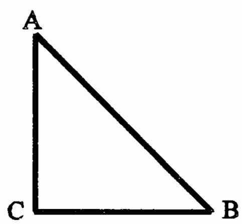
Number of Sides	Name
3	Triangle
4	Rectangle
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Features of Common Polygons

Depending upon the number of sides, each polygon has a variety of terms that describe its various parts.

Triangles

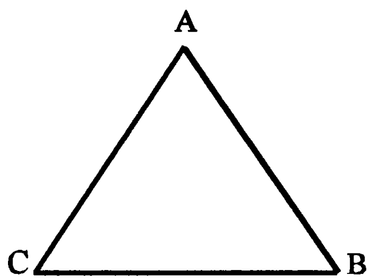
Triangles are three-sided figures with three interior angles. In ordinary (Euclidean) geometry the sum of the three interior angles is always $180°$. Typically, triangles are labeled with letters at each of the three vertices. For example:

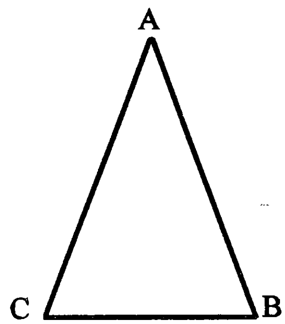


These labels then allow us to reference the entire triangle ($\triangle ABC$) and each side by its beginning and ending point. For example, the triangle above is composed of three sides, AB, BC, and AC. Furthermore, we can use these letters to define the three interior angles of the triangle. The triangle above has three interior angles; $\angle ABC$,$\angle BCA$,$\angle CAB$. In each case, the symbol describes the angle at the vertex in the middle of the three letters. The sides of triangles can also be called legs. The bottom side of a triangle is often referred to as the base and the side opposite a right angle is called the hypotenuse.

There are several different types of triangles, but each has essentially the same parts. The triangle above is a *right triangle* because $\angle BCA$ is a $90°$ angle. A right triangle also demonstrates another important relationship -- that of *perpendicular lines*. Two lines are perpendicular when their intersection forms a $90°$ angle.

In an *equilateral* triangle the sides are of equal length (congruent) and the interior angles are all equal (60°.) An *isosceles* triangle has at least two congruent sides.

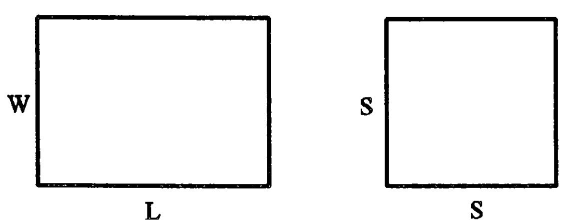




Equilateral Triangle Isosceles Triangle

Rectangles and Squares

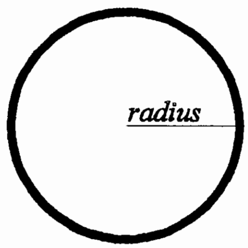
Rectangles are four sided figures with each interior angle equal to 90°. In addition, the opposite sides of a rectangle are *parallel* to each other. Therefore, the sum of the interior angles of any rectangle will be 360°. A square is a special type of rectangle with four equal sides. The important parts of rectangles are shown below.

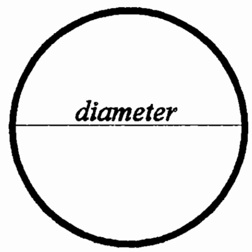


Since the sides on the square are of equal size, they are both labeled S. For the rectangle the important features are the width (W) and the length (L). These features will play an important role in the calculations of different aspects of these figures.

Circle

Circles are polygons with an infinite number of interior angles with the sum of those angles being 360°. The key parts of a circle, the radius (r) and the diameter (d), are shown below.





Perimeter of Polygons

The distance around the outside of a polygon is called the *perimeter*. In the case of a circle this distance is referred to as the *circumference*. In order to determine the perimeter or circumference you must:

1) Know the formulas for determining the perimeter for common polygons.

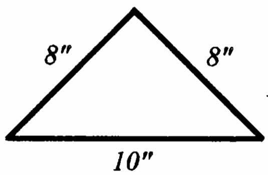
2) Make the necessary substitutions of values into the appropriate formula.

3) Make certain the answer is stated in the correct units of measure (e.g., inches, feet)

Perimeter of a Triangle

The perimeter of a triangle is determined in the same manner regardless of the type of triangle. The basic formula is: perimeter = length side #1 + length side #2 + length side #3

For example, determine the perimeter of the triangle below.

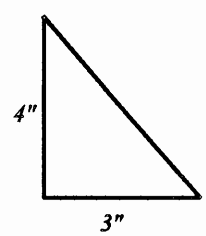


Perimeter = 8" + 8" + 10" or 26 "

The Pythagorean Theorem

One of the most useful relationships in mathematics provides a means for determining the length of the any side of right a triangle. The basic Pythagorean Theorem defines the relationship between the *hypotenuse* (the side opposite the right angle) and the other two legs. The basic relationship is: Hypotenuse^2 = (leg #1)^2 + (leg #2)^2 . This formula enables you to determine the length of different legs using basic algebraic transformations. For example, find the length of the hypotenuse of the following triangle.

$(\text{Hypotenuse})^2 = (4)^2 + (3)^2$
$(\text{Hypotenuse})^2 = 16 + 9$
$(\text{Hypotenuse})^2 = 25$
$(\text{Hypotenuse}) = 5$



Perimeter of a Rectangle

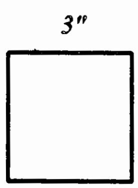
The perimeter of a rectangle is determined by summing together the length of each side. Since the square has four equal sides the formula can be written in terms of multiplication with the perimeter being equal to four times the length of a side:

Perimeter of a square = 4(s); where s is the length of any side.

Since a rectangle has two pairs of equal length sides, the formula for the perimeter of a rectangle can also be written in terms of multiplication. In this case, however, it is the twice the length plus twice the width:

Perimeter of a rectangle = 2(l) + 2(w); where l is the length and w is the width.

For example, determine the perimeter of the square below.



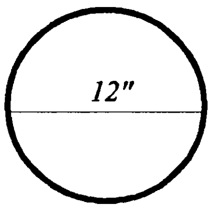
Perimeter of a Square = 4(3") or 12 inches

Circumference of a Circle

Since a circle is composed of an infinite number of straight lines it is necessary to use an irrational number (pi or π) for most calculations involving circles. In order to determine the circumference of a circle (the distance around the outside) the formula is:

$$Circumference = \pi \cdot diameter$$

For example, the circumference of the circle below is computed by multiplying the value of pi times the value of the diameter. For most purposes a value of *3.14* can be used to represent pi.



Circumference $= 3.14 * 12"$ *or* $37.7"$

Area of Polygons

A common skill in geometry is determining the area of polygons. The area is a two dimensional measure of the polygon (e.g., height and width). The key skills are:

1) Know the formulas for the area for common polygons.

2) Make the necessary substitutions of values into the appropriate formula.

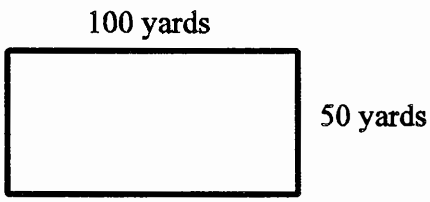
3) Make certain the answer is stated in the correct units of measure (e.g., square inches, square feet)

Area of a rectangle

The area of a rectangle is determined by multiplying the height by the width.

area = $h * w$ Where h is the height of the rectangle and w is the width.

Example: Determine the area of a football field that is 100 yards long and 50 yards wide.



Area = 100 yards * 50 yards or 5000 square yards

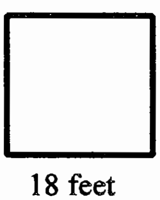
Notice that when you are computing the area of a two-dimensional (height and width) polygon the final measurement is in terms of squared units of measure (e.g., square yards.)

Area of a Square

A square is a special type of rectangle which has four equal sides. Therefore, the formula for determining the area of a square involves a single variable.

$area = s^2$ Where s is equal to the length of a side.

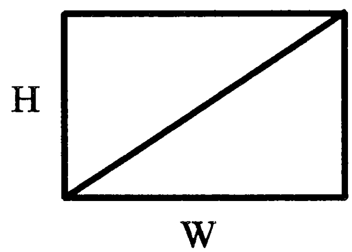
Example: Determine the area of a boxing ring that is 18 feet on each side.



Area = (18 ft) * (18 ft) equals 324 sq ft

Area of a Triangle

As shown in the diagram below, the area of a triangle is equal to 1/2 the area of a rectangle with the same height and width.



Area of a rectangle = height * width

Area of a triangle = 1/2(height * width)

For example, with a height of 5 feet and a width of 9 feet, the area of the triangle above would be equal to 1/2(5 feet * 9 feet) or 22.5 square feet.

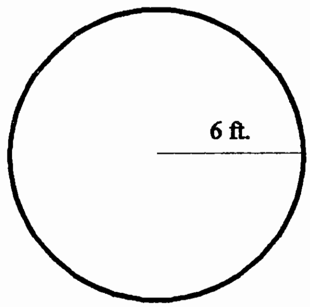
Area of a Circle

As with the circumference, determining the area of a circle requires the use of an approximate value of π (pi). For most purposes you can obtain an accurate enough answer by using the value of 3.14 to represent pi.

The area of a circle is determined by multiplying the value π times the radius (r) squared or:

$$area = \pi r^2$$

Example: Determine the area of a wrestling mat with a radius of 6 feet.



Area = (3.14) (6 ft.)(6 ft.) or 113.04 sq. ft.

Appendix A

Diagnostic Test Instructions

The brief test on the following pages is provided as a diagnostic tool to help the user to identify potential areas of study. In order to use this test, follow the three steps outlined below.

1) Take the diagnostic test.

The test consists of 20 items that tap different skill areas. In order to make the feedback from the test as useful as possible you should take the test under conditions that approximate an actual test situation. Make sure you are in a quiet place where you will not be distracted. Try to complete the test in less than 20 minutes, but be sure to complete all items. Make a check mark next to test items that take you more than 2 minutes to complete or items where you were not certain of your answer. Even if you get the correct answer, you may want to spend some time honing your skills in that area.

2) Score the diagnostic test.

Using the key on page A-6, score the diagnostic test.

3) Identify skill areas for further study.

Using the reference guide on page A-6, identify the sections where you should concentrate your study time.

Mathematics Skill Diagnostic Test

Circle the most correct answer to each of the items on the following pages. Once you have completed all the items score your answers with the key provided on page A-6. If you feel you need additional information on any of the items, refer to the corresponding chapter indicated in the scoring key.

1 A certain store stocks five different brands of frozen yogurt, and sells a pint of each for \$2.75, \$3.25, \$2.50, \$3.25, and \$3.00 respectively. What is the average price of the different types of frozen yogurt?

(A) \$2.50 (B) \$3.25 (C) \$1.95 (D) \$2.95 (E) \$3.95

2. Evaluate the following: $(\frac{3}{4})(\frac{5}{6})$?

(A) $\frac{8}{12}$ (B) $\frac{5}{8}$ (C) $\frac{8}{10}$ (D) $\frac{18}{20}$ (E) $\frac{8}{24}$

Name	Weight in Pounds
Chris	150
Anne	153
Malcolm	154
Paul	157
Sam	151

3. What is the average (arithmetic mean) weight of the five people whose weights are listed in the table above?

(A) 152 (B) 153 (C) 153 $\frac{1}{2}$ (D) 154 (E) 155

4. Which of the following is closest in value to the decimal 0.40?

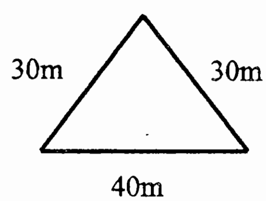
(A) $\frac{1}{3}$ (B) $\frac{12}{15}$ (C) $\frac{20}{25}$ (D) $\frac{24}{35}$ (E) $\frac{48}{60}$

5. 15 movie theaters average 600 customers per theater per day. If 6 of the theaters close down but the total theater attendance stays the same, what is the average daily attendance per theater among the remaining theaters?

(A) 500 (B) 750 (C) 1,000 (D) 1,200 (E) 1,500

6. A student's grade in a course is determined by 4 quizzes and 1 exam. If the exam counts twice as much as each of the quizzes, what fraction of the final grade is determined by the exam?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$



7. What type of triangle is shown above?

(A) equilateral (B) right (C) scalene (D) isosceles

8. Compute the perimeter of the triangle above.

(A) 100 meters
(B) 100 square meters
(C) 70 meters
(D) 70 kilometers

9. What is 18% of 40?

(A) .72 (B) 45 (C) 7.2 (D) .45 (E) 4.5

10. If $x = -3$, what is the value of the expression $x^2 + 3x + 3$?

(A) -21 (B) -15 (C) -6 (D) 3 (E) 21

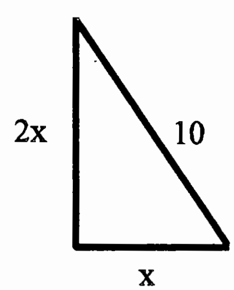
11. Which of the following is equivalent to $3x^2 + 18x + 27$?

(A) $3(x^2 + 6x + 3)$ (B) $3(x+3)(x+6)$ (C) $3(x+3)(x+3)$

(D) $3x(x+6+9)$ (E) $3x^2 + x(18+27)$

12. Compute the area of a circle with the diameter of 8 inches.

(A) 50.24 inches (B) 25.12 inches (C) 50.24 square inches
(D) 25.12 square inches (E) 64 square inches



13. In the figure above, what is the area of the right triangle?

(A) $4\sqrt{5}$ (B) 10 (C) $8\sqrt{5}$ (D) 20 (E) 40

14. If the minute hand of a clock moves 45 degrees, how many minutes of time have passed?

(A) 6 (B) 7.5 (C) 15 (D) 30 (E) 36.5

15. $(x^2)^3$ equals:

(A) x^5 (B) x^6 (C) x^{-1} (D) x (E) $x^{1.5}$

16. Multiply: $(4x^2 + 2xy - 2)(x + y)$

(*A*) $(4x - 2y - 2)^2$
(*B*) $(4x^3 - 2xy - 2x + 4xy - 4xy^3)$
(*C*) $(4x^3 + 6x^2y + 2xy^2 - 2x - 2y)$
(*D*) $(4x^{x\cdot y} - 2xy^{x\cdot y} - 2^{x\cdot y})$

17. Evaluate the following expression: $(-1 + 1) - (-3)$.

(A) -4 (B) -1 (C) 2 (D) -3 (E) 3

18. Mike's weekly salary is \$70 less than Sara's, which is \$50 more than Sue's. If Sue earns \$280 per week, how much does Mike earn per week?

(A) \$160 (B) \$260 (C) \$280 (D) \$300 (E) \$400

19. If the sum of a, b and c is twice the sum of a minus b plus a minus c, then a =

(A) $b + c$ (B) $3b + 3c$ (C) $3b - c$ (D) $2b + 3c$ (E) $-b - c$

20. $5z^2 - 5z + 4 - z(3z - 4) =$

(A) $2z^2 - z + 4$ (B) $2z^2 - 9z + 4$

(C) $5z^2 - 8z + 8$ (D) $5z^2 - 8z$

(E) $2z^2 - 5z$

Scoring Key for Mathematics Diagnostic Test

Test Item Number	Answer	Reference Chapters
1	D	4
2	B	3
3	B	4 & 5
4	A	3
5	C	4 & 6
6	D	3
7	D	7
8	A	7
9	C	4
10	D	6
11	C	6
12	C	7
13	D	7
14	B	7
15	B	3
16	C	6
17	E	3
18	B	4
19	A	6
20	A	6

Appendix B

Exercise Problems, Answers, and Solutions

Exercises for Chapter 3: Basic Arithmetic

Answers and solutions on page B-11

1. Which of the following lists three fractions in ascending order?

(A) $\frac{9}{26}, \frac{1}{4}, \frac{3}{10}$

(B) $\frac{9}{26}, \frac{3}{10}, \frac{1}{4}$

(C) $\frac{1}{4}, \frac{9}{26}, \frac{3}{10}$

(D) $\frac{1}{4}, \frac{3}{10}, \frac{9}{26}$

(E) $\frac{3}{10}, \frac{9}{26}, \frac{1}{4}$

Perform these calculations:

2. 1/2 + 5/2 =

3. 1/2 - 1/3 =

4. 1/7 + 1/9 =

5. 1/3 * 5/3 =

6. 2/3 * 4/5 =

7. 0.25 * 0.25 =

8. 14.165 + 2.315 =

Convert these fractions to decimal fractions:

9. 7/8 =

10. 5/9 =

Exercises for Chapter 4: Applied Arithmetic

Answers and solutions on page B-13

1. George drives the first 30 miles of a trip at a constant rate of 40 miles per hour. If he drives the remaining 75 miles of the trip at a constant rate of 50 miles per hour, what is his average speed for the entire trip?

   (A) 44.6 miles per hour (B) 46.7 miles per hour
   (C) 45 miles per hour (D) 55 miles per hour

2. A subway car passes 5 stations every 10 minutes. At this rate, how many stations will it pass in one hour?

   (A) 2 (B) 12 (C) 15 (D) 18 (E) 30

3. If a car travels $\frac{1}{100}$ of a kilometer each second, how many kilometers does it travel per hour?

   (A) $\frac{2}{3}$ (B) $3\frac{3}{5}$ (C) 36 (D) 72 (E) 100

4. The Lakeland factory produced two-fifths of the Acme Brick Company's bricks in 1993. If the Lakeland factory produced 1,400 tons of bricks in 1993, what was the Acme Brick Company's total output that year?

   (A) 350 tons (B) 35000 tons (C) 5600 tons (D) 3500 tons

5. At garage A, it costs \$8.75 to park a car for the first hour and \$1.25 for each additional hour. At garage B, it costs \$5.50 for the first hour and \$2.50 for each additional hour. What is the difference between the cost of parking a car for 5 hours at garage A and parking it for the same length of time at garage B?

   (A) \$1.50 (B) \$1.75 (C) \$2.25 (D) \$2.75 (E) \$3.25

6. If the ratio of boys to girls in a class is 5 to 3, which of the following could not be the number of students in the class?

(A) 32 (B) 36 (C) 40 (D) 48 (E) 56

7. If a woman earns $200 for her first 40 hours of work in a week and then is paid one-and-one-half times her regular hourly rate for any additional hours, how many hours must she work to make $230 in a week?

(A) 4 (B) 5 (C) 6 (D) 44 (E) 45

Exercises for Chapter 5: Graphs, Charts and Tables

Answers and solutions on page B-15

	Organizational Unit			
Cost Category	**Advertising**	**Communications**	**Research**	**Total**
Management	32	7	17	56
Support	50	2	12	64
Training	43	1	9	53
Total	125	10	38	173

* Cost in Millions

1. How much money was spent on Training in Communications?

   (A) 3 million (B) 10.7 million (C) 9 million (D) 1 million (E) 7 million

2. What percent of all Support money is used by Research?

   (A) 16.4 (B) 18.75 (C) 14 (D) 0.2 (E) 32

3. In what kind of work was the greatest percentage of money spent in Support?

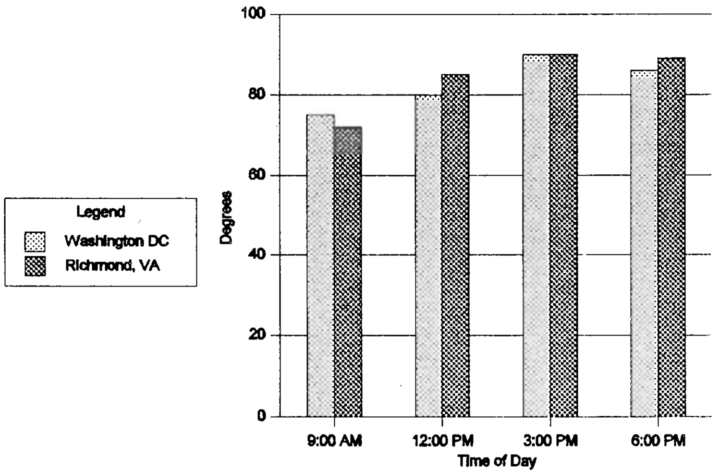
   (A) Research (B) Training (C) Advertising

   (D) Communications (E) Management

4. Within Research, what is the ratio of expenditures between Support and Training?

   (A) 3 to 1 (B) 4 to 1 (C) 4 to 3 (D) 3 to 4

Aug. Average Temperature



5. What is the coolest time of day?

(A)	5:00 pm	(B)	12:00 pm
(C)	9:00 am	(D)	3:00 pm
(E)	6:00 pm		

6. What is the least average temperature difference between cities for a particular time of day?

(A)	5 degrees	(B)	10 degrees	(C)	15 degrees
(D)	0 degrees	(E)	3 degrees		

7) What is the approximate average temperature at 12:00 pm?

(A)	90 degrees	(B)	82 degrees
(C)	80 degrees	(D)	85 degrees

Exercises for Chapter 6: Algebra

Answers and solutions on page B-16

Simplify these expressions:

1. $$\frac{x^2 - 11x + 28}{x - 7}$$

2. $$\frac{x^2 + 8x + 12}{x + 6}$$

3. $(2y + x)(2y - x)$

4. $(x^3 + x^2)(x^2 + x + 1)$

5. $(a - b)(a^2 + ab + b^2)$

6. $(a + b)(a^2 - ab + b^2)$

Solve these equations for x:

7. $2 + 3x = 0$

8. $3x = 5$

9. $\sqrt{x^2 + 2x + 1} = 2x - 5$

10. If Y is your gross income, and the government takes away $\frac{1}{4}Y - 10$ as taxes. What was your gross income if your after-tax income is \$280?

11. A grocery store ordered an equal number of cases of orange and grapefruit juice. The wholesaler delivered 30 extra cases of orange juice, making the ratio of orange juice to grapefruit juice 6:5. How many cases of juice did the store originally order?

    (A) 120 (B) 150 (C) 180 (D) 300 (E) 330

12. Between 1960 and 1970 the population of California increased by 3.5 million people. If the amount of the increase between 1970 and 1980 was 1.75 million more than the increase from 1960 to 1970, what was the total amount of increase in California's population between 1960 and 1980?

(A) 3.75 million (B) 3.5 million (C) 7 million (D) 8.75 million

13. There are enough cookies in a box to give 4 to each of 25 children, with none left over. If 5 children do not want any cookies, how many cookies can you give to each of the others?

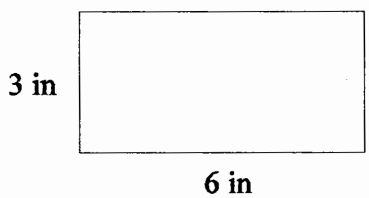
(A) 5 (B) 4.5 (C) 9 (D) 6

14. Five gallons of gasoline were drained from Tank 1 to Tank 2, and 10 gallons of gasoline were drained from Tank 1 to Tank 3. If Tank 1 originally has 10 more gallons of gasoline than Tank 3, how many more gallons of gasoline does Tank 3 now have than Tank 1?

(A) 20 (B) 0 (C) 25 (D) 15 (E) -5

Exercises for Chapter 7: Geometry

Answers and solutions on page B-20



1) Compute the perimeter of the rectangle above.

(A) 9 in. (B) 15 in. (C) 36 in. (D) 3^2 in. (E) 18 in.

2) Compute the area of the rectangle above.

(A) 36 sq in (B) 18 sq in (C) 324 sq in (D) 81 sq in

3) What is the area of a circle with a radius of 4 feet?

(A) 50.24 sq ft (B) 50.24 sq in (C) 12.56 sq ft (D) 16 sq ft

4) In a right triangle, how many degrees are in the angle opposite the hypotenuse?

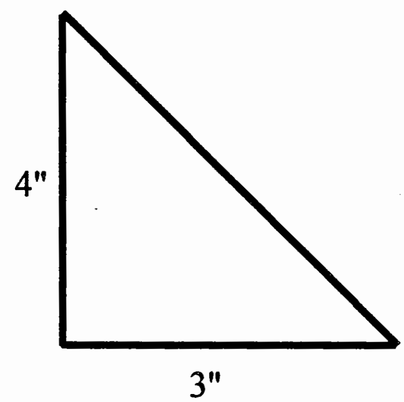
(A) 180 (B) 60 (C) 90 (D) 360 (E) 45

5) How many degrees are there for each external angle in a regular nonagon?

(A) 140 (B) 60 (C) 90 (D) 40 (E) 180

6) Which of the following types of triangles has two equal length sides?

(A) equilateral (B) equiangle (C) scalene (D) isosceles



7) What is the area of the triangle above?

(A) 12 sq in (B) 25 sq in (C) 6 sq in (D) 144 sq in

8) What is the perimeter of the triangle above?

(A) 12 in. (B) 13 in (C) 11 in (D) 50 in

Chapter 3 Answers and Solutions

Problem 1. Answer: (D)

Solution: If the fractions are converted to decimal fractions they are ordered as follows: $.25 < .30 < .35$

Problem 2. Answer: 6/2 or 3

Solution: Since the fractions have a common denominator you add the numerators and retain the denominator. Since 6/2 is an improper fraction (greater than 1) it can be simplified to the whole number 3.

Problem 3. Answer: 1/6

Solution: Since the fractions do not have a common denominator you transform the problem by multiplying each term by the equivalent of 1. Since 6 is a common multiple of 2 and 3 you can multiply 1/2 by 3/3 and 1/3 by 2/2. This results in the subtraction of 3/6 - 2/6 = 1/6.

Problem 4. Answer: 16/63

Solution: The solution is similar to the previous problem. Transform each fraction to create a common denominator then add the results.

Problem 5. Answer: 5/9

Solution: When multiplying fractions you carry out the multiplication on both numerator and denominator.

$$\frac{1}{3} \cdot \frac{5}{3} = \frac{5}{9}$$

Problem 6. Answer: 8/15

Solution: This problem involves the same procedure as problem 5.

Problem 7. Answer: .0625

Solution: Multiplication of two decimal numbers will yield a smaller decimal number.

Problem 8. Answer: 16.48

Problem 9. Answer: 875

Solution: To convert a fraction to a decimal fraction, divide the numerator by the denominator.

Problem 10. Answer: .55

Chapter 4 Answers and Solutions

Problem 1. Answer: (B) 46.7 miles per hour

Solution: Determine total distance and divide by total amount of time

Step 1: Determine time for first leg using Rate * Time = Distance
40 mph * Time = 30 miles
Time = 30 miles/40 mph or .75 hours

Step 2: Determine time for second leg
50 mph * Time = 75 miles
Time = 75 miles/50 mph or 1.5 hours

Step 3: Now you have total time of 2.25 hours and total distance of 105 miles. Dividing 105/2.25 yields 46.7 miles per hour.

Problem 2. Answer: (E) 30 stations

Solution: The subway will pass 6 times as many stations in an hour as it does in 10 minutes; therefore, 6 * 5 = 30

Problem 3. Answer: (C) 36

Solution: Multiply the number of seconds in an hour (3600) times the distance traveled in each second. 3600 * 1/100 = 36.

Problem 4. Answer: (D) 3500 tons

Solution: The 1400 tons produced by the Lakeland factory was 2/5 of the total. We can find out what 1/5 is by dividing 1400 by 2 to get 700. Since 700 is one of five parts we multiply 700 by 5 and get 3500.

Problem 5. Answer: (B) $1.75

Solution: Compute the cost of parking 5 hours in each garage then compute the difference.

Step 1: Garage A = $8.75 + (4 * $1.25) or $13.75

Step 2: Garage B = $5.50 + (4 * $2.50) or $15.50

Step 3: Garage A - B difference; $15.50 - $13.75 = $1.75

Problem 6. Answer: (B) 36

Solution: In order to yield whole number answers a ratio of 5 to 3 can only be applied to numbers that are a multiple of 8. Of the alternatives, only 36 is not a multiple of 8.

Problem 7. Answer: (D) 44

Solution:

Step 1: Determine the regular rate of pay $200/40 hours = $5.00 per hour.

Step 2: Compute overtime rate: 1.5 * $5.00 = $7.50 per hour

Step 3: Determine how long it takes to make $30 at $7.50 per hour
$30/$7.50 = 4 hours

Step 4: Add overtime hours to regular hours 40 + 4 = 44 hours

Chapter 5 Answers and Solutions

Problem 1. Answer: (D), 1 million

Solution: The answer falls at the intersection of the Communications column and the Training row.

Problem 2. Answer: (B), 18.75

Solution: Research used $12 million of the total $64 million expenditure for Support. 12 divided by 64 is 18.25.

Problem 3. Answer: (C), Advertising

Solution: Advertising spent 40% on support (50/125), followed by Research at 31.6% (12/38), and Communications at 20% (2/10).

Problem 4. Answer: (C), 4 to 3

Solution: 12 million is spent on support and 9 million is spent on training. This reduces to a ratio of 4 to 3.

Problem 5. Answer: (C), 9:00 am

Solution: Both bars at 9:00 am are below all other bars indicating the lowest temperature during that time frame.

Problem 6. Answer: (D), 0 degrees

Solution: Both Washington and Richmond average 90 degrees at 3:00 pm.

Problem 7. Answer: (B), 82 degrees

Solution: The temperature at Richmond is about 84 degrees and at Washington DC about 80 degrees. This averages to 82 degrees.

Chapter 6 Answers and Solutions

Problem 1. Answer: x - 4

Solution: Expand the numerator by factoring. One of the keys to this type of problem is to look for two numbers that when added together equal the middle number (11) and when multiplied together equal the last number (28). The numbers that meet these requirements are 4 and 7. Once the problem is factored you can cancel out like terms, in this case (x - 7). This leaves (x - 4).

(1) $\dfrac{x^2 - 11x + 28}{x - 7}$

(2) $\dfrac{(x - 4)(x - 7)}{x - 7}$

(3) $x - 4$

Problem 2. Answer: x + 2

Solution: This solution is accomplished in the same way as problem number 1. The factors of the numerator are (x + 2) and (x + 6). When the (x + 6) terms are canceled, the result is (x + 2).

Problem 3. Answer: $4y^2 - x^2$

Solution: In carrying out the multiplication of the two terms in the parentheses you get the following:

(1) $(2y + x)(2y - x)$

(2) $4y^2 - 2xy + 2xy - x^2$

(3) $4y^2 - x^2$

Problem 4. Answer: $x^5 + 2x^4 + 2x^3 + x^2$

Solution:

(1) $(x^3 + x^2)(x^2 + x + 1)$

(2) $x^5 + x^4 + x^3 + x^4 + x^3 + x^2$

(3) $x^5 + 2x^4 + 2x^3 + x^2$

Problem 5. Answer: $a^3 - b^3$

Solution:

(1) $(a - b)(a^2 + ab + b^2)$

(2) $a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$

(3) $a^3 - b^3$

Problem 6. Answer: $a^3 + b^3$

Solution:

(1) $(a + b)(a^2 - ab + b^2)$

(2) $a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$

(3) $a^3 + b^3$

Problem 7. Answer: $x = -2/3$

Solution:

(1) $2 + 3x = 0$

(2) $2 - 2 + 3x = 0 - 2$

(3) $3x = -2$

(4) $\frac{3x}{3} = \frac{-2}{3}$

(5) $x = \frac{-2}{3}$

Problem 8. Answer: $x = 5/3$

Solution:

(1) $3x = 5$

(2) $\frac{3x}{3} = \frac{5}{3}$

(3) $x = \frac{5}{3}$

Problem 9. Answer: x = 6

Solution:

(1) $\sqrt{x^2 + 2x + 1} = 2x - 5$
(2) $\sqrt{(x + 1)^2} = 2x - 5$
(3) $x + 1 = 2x - 5$
(4) $x - x + 1 = 2x - x - 5$
(5) $1 = x - 5$
(5) $1 + 5 = x - 5 + 5$
(6) $6 = x$

Problem 10. Answer: Y = \$360

Solution:

(1) $Y - [\ (\frac{1}{4})Y - \$10\ ] = \$280$
(2) $Y - (\frac{1}{4})Y + \$10 = \$280$
(3) $Y - (\frac{1}{4})Y + \$10 - \$10 = \$280 - \$10$
(4) $Y - (\frac{1}{4})Y = \$270$
(5) $(\frac{4}{4})Y - (\frac{1}{4})Y = \$270$
(6) $(\frac{3}{4})Y = \$270$
(7) $(\frac{4}{3})(\frac{3}{4})Y = (\frac{4}{3})\$270$
(8) $Y = \$360$

Problem 11. Answer: (D) 300

Solution: Let x be the number of cases of grapefruit juice received. This makes the number of cases of orange juice x + 30. We can now equate this to the ratio of 6 to 5.

(1) $\frac{6}{5} = \frac{x + 30}{x}$
(2) Cross multiply : $6x = 5(x + 30)$
(3) $6x = 5x + 150$
(4) $x = 150$
(5) Total cases $= 2x = 300$

Problem 12. Answer: (D), 8.75 million

Solution: Between 1970 and 1980 the population of California increased 1.75 million more than the increase between 1960 and 1970, or 1.75 million more than 3.5 million. So the increase between 1970 and 1980 was 1.75 + 3.5 or 5.25 million. The total growth between 1960 and 1980 therefore was,

3.5 million (1960 - 1970) + 5.25 million (1970 - 1980) = 8.75 million

Problem 13. Answer: (A), 5

Solution: First find the total number of cookies: 4 * 25 = 100. Then determine how many children will get cookies 25 - 5 = 20. Next divide the number of cookies by the number of children to get cookies, 100/20 = 5

Problem 14. Answer: (D), 15

Solution: Represent the original contents of Tank 1 as g and Tank 3 as g - 10. 5 gallons of gasoline were drained from Tank 1 to Tank 2 and another 10 gallons were drained from Tank 1 to Tank 3 for a total of 15 gallons. Now Tank 1 contains g -15 gallons of gasoline. Since Tank 3 got 10 gallons it now has g gallons of gasoline (g - 10 + 10). So Tank 3 contains 15 gallons more than Tank 1.

Chapter 7 Answers and Solutions

Problem 1. Answer: (E), 18 in.

Solution: The perimeter of a rectangle is computed by summing the length of all four sides (2h + 2w). For the example problem this is [2(3) + 2(6)] = 18.

Problem 2. Answer: (B), 18 sq in

Solution: The area of a rectangle is computed by multiplying the height times the width (area = h * w). In this example the area = 3 * 6 = 18.

Problem 3. Answer: (A), 50.24 sq ft

Solution: The area of a circle is computed by multiplying pi times the radius squared. For this example it is 3.14 * 16 = 50.24

Problem 4. Answer: (C), 90 degrees

Solution: The hypotenuse is the side opposite the right angle, which is a 90 degree angle.

Problem 5. Answer: (D), 40 degrees

Solution: A nonagon is a nine-sided polygon. Since the sum of the internal angles is 360 degrees, each angle is 40 degrees.

Problem 6. Answer: (D), isosceles

Solution: An isosceles triangle has two equal sides, an equilateral triangle has 3 equal sides.

Problem 7. Answer: (C), 6 sq in

Solution: The area of a triangle is computed by taking 1/2 of the product of the base and height. For this example it is 1/2(4 * 3) = 6

Problem 8. Answer: (A), 12 in

Solution: With a right triangle, the square of the hypotenuse is equal to the sum of the squared sides. For this example the square of the hypothenuse was the sum of 16 + 9 or 25. Taking the square root of 25 we get 5.
Adding 3 + 4 + 5 = 12.

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